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How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

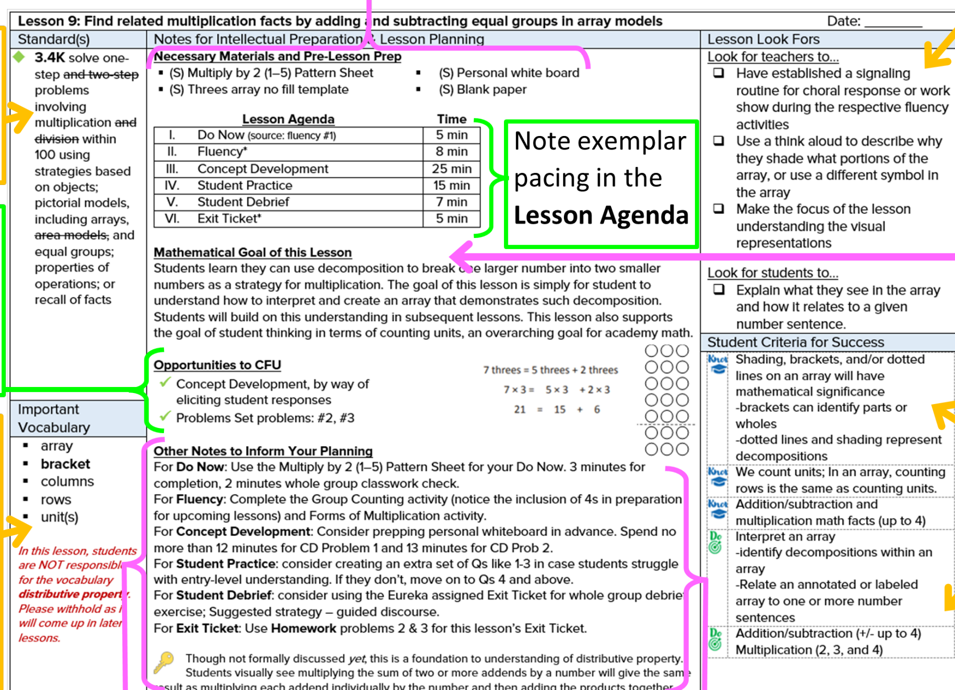
Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models — Date: ________

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models~~, and equal groups; properties of operations; or recall of facts	**Necessary Materials and Pre-Lesson Prep** ▪ (S) Multiply by 2 (1–5) Pattern Sheet ▪ (S) Personal white board ▪ (S) Threes array no fill template ▪ (S) Blank paper	Look for teachers to... ❑ Have established a signaling routine for choral response or work show during the respective fluency activities ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array ❑ Make the focus of the lesson understanding the visual representations Look for students to... ❑ Explain what they see in the array and how it relates to a given number sentence.

Lesson Agenda		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson
Students learn they can use decomposition to break one larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU
✓ Concept Development, by way of eliciting student responses
✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Important Vocabulary
▪ array
▪ **bracket**
▪ columns
▪ rows
▪ unit(s)

*In this lesson, students are NOT responsible for the vocabulary **distributive property**. Please withhold as it will come up in later lessons.*

Other Notes to Inform Your Planning
For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Student Criteria for Success
- Know: Shading, brackets, and/or dotted lines on an array will have mathematical significance -brackets can identify parts or wholes -dotted lines and shading represent decompositions
- Know: We count units; In an array, counting rows is the same as counting units.
- Know: Addition/subtraction and multiplication math facts (up to 4)
- Do: Interpret an array -identify decompositions within an array -Relate an annotated or labeled array to one or more number sentences
- Do: Addition/subtraction (+/- up to 4) Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS
Trigonometry is the study of triangles, their angles, lengths, side ratios, and more. The name comes from Greek *trigonon* meaning triangle, and *metron* meaning “measure”. The key idea in this unit is to find relationships between angles and distances. In this unit, we explore the concepts of angles and other metrics, such as speed to calculate angular velocity. Later in this unit, we will be able to take a deeper dive in the relationship between distances and angles with trigonometric ratios. We will be introduced to the unit circle and use that to develop the graphs of sine, cosine, tangent, including their reciprocal functions. Towards the end of this unit, we explore the inverse trigonometric functions and their attributes. The unit proceeds to graphing the inverse trigonometric functions as well as their transformations. The final lessons focus on the applications of these functions, e.g., solving for angles of a right triangle or the side length of a right triangle.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Texas Essential Knowledge and Skills (TEKS)	
Knowledge and Skills	**Student Expectations (SE)**
<u>(2) Functions</u> The student uses process standards in mathematics to **explore**, **describe**, and **analyze** the **attributes of functions**. The student makes connections between **multiple representations** of functions and **algebraically constructs** new functions. The student analyzes and uses functions to **model** real-world problems.	**(2.G)** Graph functions, including ~~exponential, logarithmic~~, sine, cosine~~, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. **(2.H)** Graph $\arcsin x$ and $\arccos x$ and describe the limitations on the domain. **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power~~, trigonometric, inverse trigonometric~~, and piecewise defined functions~~, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. **(2.O)** Develop and use a sinusoidal function that models a situation in mathematical and real-world problems. **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
<u>(4) Number and Measure</u> The student uses process standards in mathematics to apply **appropriate techniques**, **tools**, and **formulas** to **calculate measures** in mathematical and real-world problems.	**(4.A)** Determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems. **(4.B)** Describe the relationship between degree and radian measure on the unit circle. **(4.C)** Represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position. **(4.D)** Represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity. **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems. **(4.F)** Use trigonometry in mathematical and real-world problems, including directional bearing.

[2]Parts of standard that are crossed out are not taught in this unit but will be taught in future units.

Focus on Disciplinary Literacy	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → / Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓		✓	✓	✓				✓						✓	✓
makes use of graphic organizers	Graphic Organizers	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓			✓		✓		✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓	✓
includes strategies that support language development																	
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓									✓				
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share		✓	✓	✓	✓											
includes sentence stems to support students with explanations	- Sentence Stems		✓	✓				✓						✓	✓		
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓		✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics					✓	✓	✓							✓	✓	✓
includes websites or equipment that enhances the lesson	Technological Support	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			
content can be presented in different forms																	
uses hands-on tools or manipulatives to represent the math	- Concrete		✓				✓										
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

ROADMAP

AT A GLANCE: Unit 4 - Trigonometric Functions and Graphs			
Day	Date	Lesson	Lesson Title
1		1	Angles and Their Measures
2		2	Radian Measure
3		3	Arc Length & Area of a Sector
4		4	Linear and Angular Velocity
5		5	Six Trig Functions of Acute Angles
6		6	The Unit Circle
7		7	Reference Angles and Trig Functions of Any Angle
8		8	Graphing Sine and Cosine Functions
9		9	Modeling with Sine and Cosine
10		10	Graphing Tangent and Cotangent Functions
11		11	Graphing Secant and Cosecant Functions
12		12	Graphing and Evaluating Inverse Trigonometric Functions
13		13	Graphing Transformations of Inverse Trig Functions
14		14	Compositions of Inverse Trig Functions
15		15	Right Triangle Trigonometry Word Problems (Introduction)
16		16	Right Triangle Trigonometry Word Problems (Intermediate)
17			*Unit 4 Success Day Alpha – Review topics based on your data*
18			*Unit 4 Success Day Beta – Unit Assessment Review*
19			**End of Unit 4 Assessment**

Date: ________

Lesson 1: Angles and Their Measures

Standard(s)

- **(4.C)** Represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position.

Important Vocabulary

- Angle
- Vertex
- Initial Side
- Terminal Side
- Standard Position
- Positive Angle
- Negative Angle
- Coterminal Angles

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- GeoGebra (optional)
- Document camera
- Colored pencils
- Student laptops (optional)

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Student Practice (15 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In the first lesson of the unit, students are introduced to the idea of angles as an object is rotated around a circle. Students will learn that an angle in standard position is one with its initial side on the positive x-axis and vertex at the origin. Positive angles are found when the terminal angle moves in a counterclockwise direction, while negative angles are found in the clockwise direction.

Opportunities to CFU

- ✓ In which direction are positive (and negative) angles measured? (INM)
- ✓ How are coterminal angles similar? (INM)

Other Notes to Inform Your Planning

- The concepts in this lesson will be very important as students learn to graph trigonometric functions later in the unit.
- In Unit 5, students will further discuss angles in the context of oblique triangle.

Focus on Disciplinary Literacy

INM

Lesson Look Fors

Look for teachers to…

- ❑ Ask probing questions on which direction angles can be measured.

Look for students to…

- ❑ Draw angles with the prescribed colored pencil(s).
- ❑ Measuring positive angles in the counterclockwise direction.
- ❑ Measuring negative angles in the clockwise direction.
- ❑ Give at least two coterminal angles given one angular measure.

Student Know/Do Chart

- Know: Positive angles are found when the terminal side of an angle has been rotated in a counterclockwise direction, while negative angles are found when the terminal side has been rotated in a clockwise direction.
- Know: Coterminal angles are found by adding or subtracting a full rotation of a circle.
- Do: Draw and measure angles in standard position.
- Do: Calculate coterminal angles.

Date: ________

Lesson 2: Radian Measure

Standard(s)

- **(4.B)** Describe the relationship between degree and radian measure on the unit circle.
- **(4.C)** Represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position.

Important Vocabulary

- Arc
- Degree
- Radian

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (28 min)
- Student Practice (12 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson is used to derive the meaning of a radian measure. Students will learn to convert from radians to degrees, multiplying the given radians by $\frac{180°}{\pi}$, and to convert from degrees to radians, by multiplying the given degrees by $\frac{\pi}{180°}$. Students will be expected to calculate and graph coterminal angles given either degrees or radians.

Opportunities to CFU

- ✓ What is the relationship between the radius and circumference of a circle? (Do Now)
- ✓ What is π in degrees? (INM)
- ✓ How do you convert between radian and degrees? (mid-INM)

Other Notes to Inform Your Planning

- Students will also learn the basic radian measures from the first quadrant and multiply those to find other values in the circle. This will be helpful in lesson 5 when students are building the unit circle.
- The Do Now is the key component to this lesson. It drives the exploration on how radians are calculated.
- In AP Calculus, students only use radians, not degrees.

Lesson Look Fors

Look for teachers to…

- ❑ Engages students in group work beginning of the INM.
- ❑ Probes students with the guiding questions on the lesson that leads to a generalization on converting between radians and degrees.

Look for students to…

- ❑ Group work in the beginning of the INM by completing #1-6.
- ❑ Generalize the conversion between radians and degrees.

Student Know/Do Chart

- Know: Angles can be measured using degrees or radians.
- Know: To convert from radians to degrees, multiply the given radians by $\frac{180°}{\pi}$.
- Know: To convert from degrees to radians, multiply the given degrees by $\frac{\pi}{180°}$..
- Do: Convert angular measures from degrees to radians.
- Do: Convert angular measures from radians to degrees.

Date: ________

Lesson 3: Arc Length & Area of a Sector

Standard(s)

- **(4.D)** Represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity.

Important Vocabulary

- **Arc Length**
- **Central Angle**
- **Area of a Sector**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (24 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will investigate the relationship between a central angle and its intercepted arc and area. Students will use prior knowledge from the previous lesson to solve problems involving arc length and area of a sector.

Opportunities to CFU

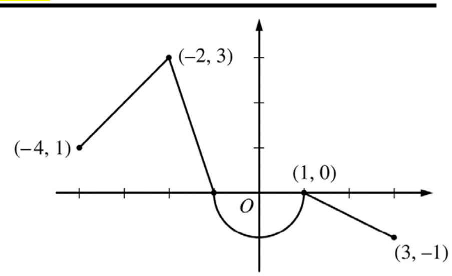
- ✓ How does circumference play a role in finding arc length? (Beginning of INM)
- ✓ How does the central angle affect the area of a sector? (mid-INM)

Focus on Disciplinary Literacy

Student Practice

Other Notes to Inform Your Planning

- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the **2012 AP Calculus AB/BC released exam**.



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

Lesson Look Fors

Look for teachers to…

- ❑ Reviews basic characteristics of a circle.
- ❑ Use the concept of a circumference of a circle to develop the formulas for arc length and area of sector.

Look for students to…

- ❑ Derive the formulas for arc length and area of a sector from proportions of a circumference of a circle.

Student Know/Do Chart

- Know: Proportions can be used to solve for arc length or area of sector because they represent a proportional part to the entire circumference or area, respectively.
- Do: Calculate arc length.
- Do: Calculate area of a sector.

Date: ________

Lesson 4: Linear and Angular Velocity

Standard(s)

- **(4.D)** Represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity.

Important Vocabulary

- **Linear Speed**
- **Angular Speed**

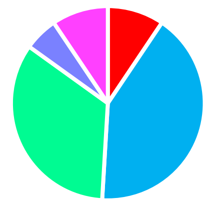
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (22 min)
- Student Practice (18 min)
- Debrief (3 min)
- Exit Ticket (5 min)



Mathematical Goal of this Lesson

In this lesson, students will be expanding their understanding of arc length from the prior lessons. Students will be examining how speed is associated with traveling along an arc length.

Opportunities to CFU

- ✓ How do linear and angular velocity compare?
- ✓ How can you find angular velocity given linear velocity?
- ✓ How does the length of a radius affect linear and angular velocity?

Other Notes to Inform Your Planning

- As this lesson is based heavily on the last lesson, this one does not need to be teacher driven. Instead, students will use their prior knowledge of velocity in real world situations in order to find connections between linear and angular velocity.
- Units are incredibly important throughout this lesson and must be stated in each question. Whenever students make a conversion, the goal is to multiply a given amount and unit by a fraction that is equal to 1. If the denominator of the fraction is equal to the given numerator unit, then they cancel.

Focus on Disciplinary Literacy

INM

Lesson Look Fors

Look for teachers to…

- ❑ Accessing prior knowledge on arc length and area of a sector at the beginning of the INM.
- ❑ Embeds context into the lessons (unit conversion, units, interpreting them. etc.).

Look for students to…

- ❑ Writing complete sentences in the opening activity of the INM.
- ❑ Writing units of measure in their workbook.
- ❑ Interpreting numerical values in context.

Student Know/Do Chart

- Know: Linear velocity is the change arc length over time.
- Know: Angular velocity is the change of the central angle over time.
- Do: Calculate linear velocities given a set of parameters.
- Do: Calculate angular velocities given a set of parameters.

Date: ________

Lesson 5: Six Trig Functions of Acute Angles

Standard(s)

- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
- **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems.

Important Vocabulary

- Sine
- Cosine
- Tangent
- Cosecant
- Secant
- Cotangent
- SOH CAH TOA

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (26 min)
- Student Practice (12 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

Yesterday's lesson discussed angular velocity, the rate of change of the angle as it rotates around a circle. By the end of the class, students should be able to name the value for all six trigonometric functions given one trig ratio, or the coordinate point of the intersection of the terminal side and the incepted circle (for acute angles only).

Opportunities to CFU

- ✓ How are the sine, cosine, tangent, cotangent, secant, and cosecant of a given angle θ calculated?
- ✓ How are sine, cosine, and tangent functions related to their reciprocal functions, cosecant, secant, and cotangent, respectively?

Other Notes to Inform Your Planning

- Students often mistake the input from the output of trig functions. Having the parts of the trig function equation displayed can be helpful for students to refer to.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam.

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

Lesson Look Fors

Look for teachers to…

- ❑ Access prior knowledge from Geometry to the Do Now using the Pythagorean Theorem.
- ❑ Describes the trigonometric functions as the side ratios to a right-angle triangle.

Look for students to…

- ❑ Complete the Do Now as much as they can and name at least one exponent rule from the Do No activity.
- ❑ Working out the proofs for each property individually or in groups.

Student Know/Do Chart

- Know: A trigonometric function models their corresponding side length ratios of a triangle inscribed in a circle.
- Know: The SOH is sine of theta equal opposite over hypotenuse. The CAH is cosine of theta equals adjacent over hypotenuse. The TOA is tangent of theta equals opposite over adjacent.
- Know: The other three trig function are reciprocals of sine, cosine, and tangent.
- Do: Compute trigonometric functions involving acute angles.

Date: ________

Lesson 6: The Unit Circle

Standard(s)

- **(4.A)** Determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.
- **(4.C)** Represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position.

Important Vocabulary

- **Unit Circle**

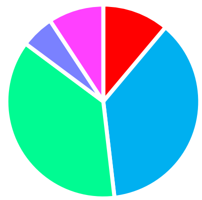
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)



Mathematical Goal of this Lesson

Today's lesson is pivotal in this unit as it walks students through the derivation of the unit circle. Students will also practice finding all six trigonometric values of the special right triangles. By the end of this lesson, students will have a completed unit circle.

Opportunities to CFU

- ✓ What connections from the special triangles and the values of the unit circle can be made?
- ✓ What is the unit circle and how is it related to the six trigonometric functions?
- ✓ How are the sine, cosine, tangent, cotangent, secant, and cosecant of a given angle θ calculated?

Other Notes to Inform Your Planning

- While this lesson is mainly exploratory for the students, the first page will be teacher-led to make sure there are no misconceptions before students begin the group work.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. This FRQ problem requires students to solve the particular solution involving logarithms after integration.

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

Lesson Look Fors

Look for teachers to…

- ❑ Use the special triangles from the Do Now to derive the values of the unit circle for Quadrant I.
- ❑ Uses the concept of symmetry to locate the corresponding values of the unit circle from Q1 for the rest of the quadrants.

Look for students to…

- ❑ Students create their own unit circle with paper plates.

Student Know/Do Chart

- Know: The unit circle uses reference triangles and quadrantal angles to evaluate trigonometric functions for all integer multiples of 30° or 45°.
- Know: These special values are wrapped to the 16 special points shown on the unit circle (a reference tool).
- Do: Calculate any trigonometric ratio from the angles of the unit circle.

Date: ________

Lesson 7: Reference Angles & Trig Functions of Any Angle

Standard(s)

- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
- **(4.A)** Determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.
- **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems.

Important Vocabulary

- **Reference Angle**
- **Angle in Standard Position**
- **All Students Take Calculus**

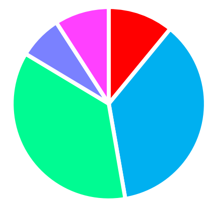
Notes for Intellectual Preparation & Lesson Planning

<u>**Necessary Materials and Pre-Lesson Prep**</u>

- Graphing calculators
- Document camera

Lesson Structure:

Do Now (6 min)	
INM (20 min)	
Student Practice (20 min)	
Debrief (4 min)	
Exit Ticket (5 min)	



<u>**Mathematical Goal of this Lesson**</u>

Today's lesson extends the analysis of the Unit Circle onto a circle of any radius. By the end of the lesson, students will be able to calculate all six trigonometric ratios for any given angle on or beyond the unit circle.

<u>**Opportunities to CFU**</u>

- ✓ How can you check if you have an erroneous solution? (INM)
- ✓ When dealing with multiple logs in an equation, what can we apply? (INM)
- ✓ How does the 1-1 property assist in solving logarithmic equations? (INM)

<u>**Other Notes to Inform Your Planning**</u>

- Today's content is a combination of the last few lessons. Starting with a review of topics will help students.
- Guide students through the trig functions and their relative side length ratios, then walk students through the Quadrant Investigation by relying heavily on student responses.

Focus on Disciplinary Literacy

INM

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

Lesson Look Fors

<u>Look for teachers to…</u>

- ❑ Review the Pythagorean triples in the Do Now.
- ❑ Uses All Students Take Calculus to show signs of trig functions.

<u>Look for students to…</u>

- ❑ Creates a graphic organizer on the signs of trig functions.
- ❑ Uses All Students Take Calculus graphic organizer to solve problems in the INM and Student Practice.

Student Know/Do Chart

Know: A circle of any radius length can follow the same functional analysis techniques to calculate the six trigonometric function values.

Do: Apply properties of logs to re-write and solve logarithmic equations.

Do: Determine which solutions involving logarithms are erroneous.

Date: ________

Lesson 8: Graphing Sine and Cosine Functions

Standard(s)

- **(2.G)** Graph functions, including ~~exponential, logarithmic~~, sine, cosine, ~~rational, polynomial, and power functions~~ and their transformations, including $af(x), f(x)+d, f(x-c), f(bx)$ for specific values of $a, b, c,$ and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power~~, trigonometric, inverse trigonometric, ~~and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.
- **(4.A)** Determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.

Important Vocabulary

- **Sine**
- **Cosine**
- **Amplitude**
- **Midline**
- **Phase Shift**
- **Period**
- **Periodic Function**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (7 min)
- INM (30 min)
- Student Practice (12 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson students will be graphing and analyzing the sine and cosine functions. Students will be asked to name the characteristics of given equations or functions, as well write equations of functions with given characteristics.

Opportunities to CFU

- ✓ How do the values of the unit circle help us graph sine and cosine?
- ✓ How are transformations of sinusoidal graphs similar and different from the previous units?

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy
Student Practice

- The Do Now starts the lesson by using the coordinates of the Unit Circle to plot the coordinate points of both functions. It is important for students to see that the functions should be represented by a smooth curve, without sharp corners. Students will also see that cosine is a horizontal translation of the sine function by 90° or $\frac{\pi}{2}$.
- Students will then use Desmos.com to investigate the equation of sine (or cosine) $f(x) = A \cdot \sin\big(B(x+C)\big) + D$ by adjusting the values for A, B, C, and D. They will discover that A represents the amplitude, B is used to calculate the period of the function, C represents the horizontal translation, and D represents the vertical translation or midline.

2012 Released AP Calculus AB Exam FRQ #6

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

   (a) For $0 \le t \le 12$, when is the particle moving to the left?

Lesson Look Fors

Look for teachers to…

- ❑ Released students in the Do Now to complete two tables.
- ❑ Facilitates a discussion of how both sine and cosine are similar and different (INM).
- ❑ Names the key attributes to a sinusoidal function.

Look for students to…

- ❑ Use technology (or the unit circle) to complete the tables in the Do Now.
- ❑ Uses Desmos to explore how $A, B, C,$ and D affect the scale and location of a sinusoidal function.

Student Know/Do Chart

- Know: The parent function for a sinusoidal graph can be derived from the values of the unit circle.
- Know: Key attributes to a sinusoidal graph.
- Know: How $A, B, C,$ and D affect the scale and location of a sinusoidal function.
- Do: Graph the $y = \sin x$ and $y = \cos x$.
- Do: Graph transformations of sinusoidal graphs.

Date: ________

Lesson 9: Modeling with Sine and Cosine Functions

Standard(s)

- **(2.O)** Develop and use a sinusoidal function that models a situation in mathematical and real-world problems.

Important Vocabulary

- **Sinusoid**
- **Model**

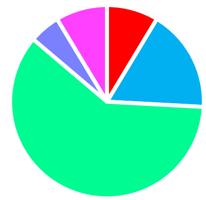
Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>

- Graphing calculators
- Computers for each student
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (10 min)
- Student Practice (35 min)
- Debrief (3 min)
- Exit Ticket (5 min)



<u>Mathematical Goal of this Lesson</u>

Today's lesson is intended to deepen the student's understanding of the sine and cosine function using real-world situations. By the end of this lesson, students will start on an infographic using Canva.com. If students don't finish it in class, it can be for Homework.
If technology is not available, proceed to the Exit Ticket.

<u>Opportunities to CFU</u>

- ✓ How can you identify the period from the data?
- ✓ How can you identify the amplitude from the data?

<u>Other Notes to Inform Your Planning</u>

- Graphing calculators can be used throughout the lesson to verify the functions. However, students may struggle with the proper windows to view the graph. Remind them to look at the domain and range of the situation and use those as guidance.

Lesson Look Fors

<u>Look for teachers to…</u>

- ❑ Warm-call students to share their responses in the Do Now.
- ❑ Have a student read the introduction paragraph, then walk students through the questions.

<u>Look for students to…</u>

- ❑ Creating a scatterplot of the data in the INM.
- ❑ If technology is present, students are creating an infographic using Canva.com.

Student Know/Do Chart

- Certain phenomena can be modeled by a sinusoidal curve.
- Identify amplitude and midline from a data set.
- Identify period from a data set.
- Identify the phase shift from a data set.

Date: ________

Lesson 10: Graphing Tangent and Cotangent Functions

Standard(s)

- **(2.G)** Graph functions, including ~~exponential, logarithmic~~, sine, cosine, ~~rational, polynomial, and power functions~~ and their transformations, including $af(x), f(x)+d, f(x-c), f(bx)$ for specific values of $a, b, c,$ and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power~~, trigonometric, inverse trigonometric, ~~and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.

Important Vocabulary

- **Tangent**
- **Cotangent**
- Period
- Asymptotes

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (7 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson is used to investigate the tangent and cotangent functions. When working with the side ratios on the Unit Circle, students already have used the idea that $tan\theta = \frac{sin\theta}{cos\theta}$, and $\cot\theta = \cos\theta\,/\,\sin\theta$. By the end of class, students should be able to compare and contrast characteristics of tangent and cotangent graphs.

Opportunities to CFU

- ✓ How is the period defined for tangent and cotangent?
- ✓ Why does tangent (or cotangent) not have an amplitude?

Other Notes to Inform Your Planning

- In the Do Now, consider using a calculator to substitute larger values of θ in $\tan\theta$. Students should see that as $\theta \to 90°\left(\text{or}\,\frac{\pi}{2}\right)$, $\tan\theta \to \infty$.
- The Do Now and INM has students build the graph of tangent using values from the unit circle. In the Student Practice, students will repeat this same process and create the graph of cotangent and examine some of its key characteristics and how they relate to tangent.

Focus on Disciplinary Literacy
INM

Lesson Look Fors

Look for teachers to…

- ❑ Explore the local behavior of $\tan\theta$ as $\theta \to 90°$. Teacher can employ technology for values of θ not on the unit circle.
- ❑ Encourages discourse/group work where students can discover how the period for tangent is half of the other trig functions.

Look for students to…

- ❑ Generalizes the sign of tangent and cotangent for each quadrant.
- ❑ Identifies asymptotes of tangent and cotangent.

Student Know/Do Chart

- Know: Tangent and cotangent have a period of π.
- Know: Tangent and cotangent are not bounded.
- Do: Graph tangent and cotangent, and their transformations.

Date: ________

Lesson 11: Graphing Secant and Cosecant Functions

Standard(s)

- **(2.G)** Graph functions, including ~~exponential, logarithmic,~~ sine, cosine, ~~rational, polynomial, and power functions~~ and their transformations, including $af(x), f(x)+d, f(x-c), f(bx)$ for specific values of $a, b, c,$ and d, in mathematical and real-world problems.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power,~~ trigonometric, inverse trigonometric, ~~and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.

Important Vocabulary

- **Cosecant**
- **Secant**
- Period
- Asymptotes

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Colored pencils
- Document camera

Lesson Structure:

- Do Now (6 min)
- INM (20 min)
- Student Practice (18 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

Today's lesson is an investigation of the secant and cosecant functions. Since cosecant is the reciprocal of the sine function, students will use the graph of the sine function to identify vertical asymptotes and critical points of the cosecant graph.

Opportunities to CFU

- ✓ How is the period defined for cosecant and secant?
- ✓ What are the domains to secant and cosecant?
- ✓ How is the amplitude related to the graphs of secant and cosecant?

Other Notes to Inform Your Planning

- Vertical asymptotes are found when the denominator of a function is equal to 0, so since $csc\theta = \frac{1}{sin\theta}$, when sine equals 0, cosecant is undefined. The maximum and minimum values of sine are 1 and -1. And the reciprocal of both those values are the same. Therefore, the maximum value of 1 because the relative minimum of secant, and the minimum value of -1 becomes the relative maximum of secant. As a class, students will comply the important information about the cosecant function.
- During the Student Practice, students will follow a similar process to derive the secant function from the graph of the cosine function. By the end of class, students should be able to compare and contrast characteristics of secant and cosecant graphs.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Use the key characteristics of sine to model the graph of cosecant (INM).
- ❑ Same for cosine with secant (Student Practice).

Look for students to…

- ❑ Graph sine and cosine in the Do Now.
- ❑ Making connections to the graph of sine with the graph of cosecant (reciprocal).

Student Know/Do Chart

- Know: Sine and cosecant are reciprocals. So is cosine and secant.
- Know: The graphs of secant and cosecant can be modeled by the reciprocal values of sine and cosine.
- Do: Graph the secant and cosecant graphs.
- Do: Identify key attributes to the graphs of secant and cosecant.

Date: ________

Lesson 12: Graphing and Evaluating Inverse Trig Functions

Standard(s)

- **(2.H)** Graph arcsinx and arccosx and describe the limitations on the domain.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power, trigonometric~~, inverse trigonometric, ~~and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.
- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

Important Vocabulary

- One-to-One
- Arcsine
- Arccosine
- Arctangent

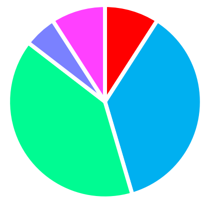
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Desmos (optional)
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Student Practice (22 min)
- Debrief (3 min)
- Exit Ticket (5 min)



Mathematical Goal of this Lesson

This lesson begins with a review of the graphs of sine and cosine and how we can make domain restrictions to the trigonometric functions to find their inverse. By the end of the lesson, students will compute inverse trigonometric values.

Opportunities to CFU

- ✓ What are the requirements for a function to have an inverse that is a function?
- ✓ How do we restrict the domain of sine and cosine so that they have an inverse that is a function?

Other Notes to Inform Your Planning

- In the next lesson, students will need to compute inverse trigonometric values involving compositions of inverse trig functions.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. This FRQ problem requires students to solve the particular solution involving logarithms after integration.

Focus on Disciplinary Literacy

INM/Student Practice

If $f(x)=\sin^{-1}x$ then $f'\left(\frac{\sqrt{3}}{2}\right)=$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{4}{7}$ D) 2

Lesson Look Fors

Look for teachers to…

- ❑ Connect prior knowledge of sine and cosine to create their respective inverses over a restricted domain.

Look for students to…

- ❑ Draw the graphs of sine and cosine and restrict its domain to satisfy the one-to-one property.
- ❑ Use the unit circle to help compute inverse trigonometric values.

Student Know/Do Chart

- Know: Some functions need to have a domain restriction so that its inverse function can be defined.
- Know: The domain and range for a function interchange when deriving its inverse.
- Do: Define the inverses of sine and cosine.
- Do: Compute inverse trigonometric values.

Date: ________

Lesson 13: Graphing Transformations of Inverse Functions

Standard(s)

- **(2.H)** Graph arcsinx and arccosx and describe the limitations on the domain.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power, trigonometric~~, inverse trigonometric, ~~and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.
- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

Important Vocabulary

- One-to-One
- Arcsine
- Arccosine
- Arctangent

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Student Practice (18 min)
- Debrief (2 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lessons begins with review of the previous lesson and will be the last lesson transformations of functions. By the end of the lesson, students will be able to Graph and analyze transformations of inverse trigonometric functions.

Opportunities to CFU

- ✓ How are the rules of transformations different and similar for different parent functions?
- ✓ In what order should transformations be applied?

Other Notes to Inform Your Planning

- If students are struggling with transformations, remind them that they follow the same rules as transforming other functions.
- In AP Calculus AB/BC, students will apply the knowledge and skills in Pre-Calculus to solve this multiple-choice problem from the 2012 AP Calculus AB released exam. This FRQ problem requires students to solve the particular solution involving logarithms after integration.

If $f(x)=\sin^{-1}x$ then $f'\left(\frac{\sqrt{3}}{2}\right)=$

A) $\frac{\pi}{6}$ B) $\frac{\pi}{3}$ C) $\frac{4}{7}$ D) 2

Lesson Look Fors

Look for teachers to…

- ❑ Connects prior knowledge of transformations of functions to inverse trig functions.

Look for students to…

- ❑ Graphing transformations of inverse functions.

Student Know/Do Chart

Know: The rules of transformations of inverse trig functions are the same as other functions.

Do: Graph transformations of inverse functions given a set of parameters.

Date: ________

Lesson 14: Composition of Inverse Trig Functions

Standard(s)

- **(2.H)** Graph arcsinx and arccosx and describe the limitations on the domain.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power, trigonometric~~, inverse trigonometric~~, and piecewise defined functions, including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/ decreasing.
- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.

Important Vocabulary

- One-to-One
- Arcsine
- Arccosine
- Arctangent

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (7 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

This lesson is structured to build an understanding of how domain restrictions impact the composition of a trig function and an inverse trig function. By the end of the lesson, students will be able to evaluate compositions of inverse trigonometric functions.

Opportunities to CFU

✓ What do you need to consider before evaluating a composition of an inverse trig function?

Other Notes to Inform Your Planning

- The Do Now begins with a refresher of evaluating trig expressions, both unit circle values as well as right triangle trig, aka SOHCAHTOA..
- The INM transitions this line of thought to trig functions in general along with their restricted one-to-one domains. Students derive the rules for composite functions. Leading to the concept that there is only one output for every inverse trig expression, and it must lie in the restricted domain. Students then explore how to evaluate compositions when the values are not on the unit circle (by drawing a triangle).

Lesson Look Fors

Look for teachers to...

- ❑ Breaks down the evaluation of composites of inverse trig functions by performing the evaluation of the inside, then the outside.
- ❑ Considers situations where the unit circle may not be useful.

Look for students to...

- ❑ Use the unit circle as a guide to evaluate composites in this lesson.
- ❑ Draw models for problems for which the unit circle does not help.

Student Know/Do Chart

- Know: Evaluating composite of inverse trig functions are similar to Unit 1.
- Know: Some problems may be more useful drawing a model.
- Do: Evaluate the composition of inverse trig functions.

Date: ________

Lesson 15: Right Triangle Trigonometry Word Problems (Introduction)

Standard(s)

- **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems.

Important Vocabulary

- Angle of Elevation
- Angle of Depression

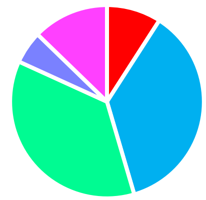
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Computers for Desmos
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Student Practice (20 min)
- Debrief (3 min)
- Exit Ticket (7 min)



Mathematical Goal of this Lesson

This lesson is structured for students to use various solving techniques to solve word problems involving right triangles. By the end of the lesson, students will be able to solve real-world problems using right triangle trigonometry.

Opportunities to CFU

- ✓ What is the difference between an angle of elevation and an angle of depression?
- ✓ What trigonometric function can be used to model this scenario?

Other Notes to Inform Your Planning

- The Do Now begins with a refresher of inverse trig and solving right triangles using SOHCAHTOA. There are then a series of seemingly random equations to solve which flush out the idea that functions are "undone" by their inverses.
- The INM transitions this line of thought to trig functions. If students want to solve for the angle of a trig expression, they can utilize an inverse trig function to do so. From here the rest of the class is practicing solving word problems using Pythagorean theorem, trig or inverse trig to find unknowns.

Focus on Disciplinary Literacy
INM

4. A 15 foot ladder is sliding down a building at constant rate of 2 feet per minute. How fast is the base of the ladder moving away from the building when the base of the ladder is 9 feet from the building?

(A) $-\frac{64}{9}\frac{ft}{min}$ (B) $\frac{2}{3}\frac{ft}{min}$ (C) $\frac{3}{2}\frac{ft}{min}$

(D) $\frac{8}{3}\frac{ft}{min}$ (E) $\frac{7}{2}\frac{ft}{min}$

Lesson Look Fors

Look for teachers to…

- ❑ Performs a quick review of SOHCAHTOA as it will be used in this lesson.

Look for students to…

- ❑ Identify which trigonometric function can be used to model a problem.
- ❑ Apply inverse trig to solve a right triangle.

Student Know/Do Chart

- Know: Some situations can be modeled by a right triangle.
- Know: We can use a trigonometric function to model a right triangle.
- Do: Draw a model and identify which trigonometric function best models the situation.
- Do: Applies inverse trig to solve a right triangle.

Date: ________

Lesson 16: Right Triangle Trigonometry Applications (Intermediate)

Standard(s)

- **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems.

Important Vocabulary

- Angle of Elevation
- Angle of Depression

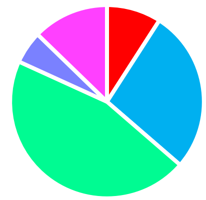
Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- Graphing calculators
- Document camera

Lesson Structure:

- Do Now (5 min)
- INM (15 min)
- Student Practice (25)
- Debrief (3 min)
- Exit Ticket (7 min)



Mathematical Goal of this Lesson

This lesson is an extension of the previous topic.

Opportunities to CFU

- ✓ What is the difference between an angle of elevation and an angle of depression?
- ✓ What trigonometric function can be used to model this scenario?

Other Notes to Inform Your Planning

- The Do Now begins with a refresher of inverse trig and solving right triangles using SOHCAHTOA. There are then a series of seemingly random equations to solve which flush out the idea that functions are "undone" by their inverses.
- The INM transitions this line of thought to trig functions. If students want to solve for the angle of a trig expression, they can utilize an inverse trig function to do so. From here the rest of the class is practicing solving word problems using Pythagorean theorem, trig or inverse trig to find unknowns.

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4. A 15 foot ladder is sliding down a building at constant rate of 2 feet per minute. How fast is the base of the ladder moving away from the building when the base of the ladder is 9 feet from the building?

(A) $-\frac{64}{9}\ \frac{ft}{min}$ (B) $\frac{2}{3}\ \frac{ft}{min}$ (C) $\frac{3}{2}\ \frac{ft}{min}$

(D) $\frac{8}{3}\ \frac{ft}{min}$ (E) $\frac{7}{2}\ \frac{ft}{min}$

Lesson Look Fors

Look for teachers to…

- ❑ Performs a quick review of SOHCAHTOA as it will be used in this lesson.

Look for students to…

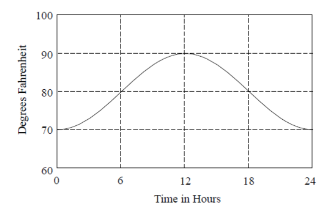
- ❑ Identify which trigonometric function can be used to model a problem.
- ❑ Apply inverse trig to solve a right triangle.

Student Know/Do Chart

- Know: Some situations can be modeled by a right triangle.
- Know: We can use a trigonometric function to model a right triangle.
- Do: Draw a model and identify which trigonometric function best models the situation.
- Do: Applies inverse trig to solve a right triangle.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
(2.G) Graph functions, including ~~exponential, logarithmic~~, sine, ~~cosine, rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.	**Concepts:** **Transformations on Function** • Vertical stretch/shrink: $af(x), a>0$ • Vertical reflection: $-f(x)$ • Horizontal reflection: $f(-x)$ • Translation (left/right): $f(x-c), c\neq 0$ • Translation (up/down): $f(x)+d, d\neq 0$ • Horizontal stretch/compression: $f(bx), b>0$ **Types of Transformations** • A **rigid** transformation does not change the shape or size of the preimage (function). • A **nonrigid** transformation will change the size but not the shape of the preimage (function). **Content:** Including, but not limited to: • Identifying the types of transformations on a function given multiple representations; graphs, tables of values, and the function expression. • Transformations of a function will follow the order of operations; grouping, exponents, multiplication, then addition.	*Below are the content and skill connections between the Algebra21 TEKS.* **Algebra 2 TEKS** When $f(x)$ is replaced including $af(x), f(x)+d, f(x-c), f(bx)$ for specific positive and negative values of a, b, c, and d, determine the effect on the: ○ **(4.C)** graph of $f(x)=\sqrt{x}$ ○ **(5.A)** graph of $f(x)=b^x$ and $f(x)=log_b\, x$ where b is 2, 10, and e ○ **(6.A)** graph of $f(x)=x^3$ and $f(x)=\sqrt[3]{x}$ ○ **(6.C)** graph of $f(x)=\|x\|$ ○ **(6.G)** graph of $f(x)=\frac{1}{x}$ **AP Calculus AB 1998 Released FRQ #5 Part a** **1998 Calculus AB Scoring Guidelines** 5. The temperature outside a house during a 24-hour period is given by $F(t)=80-10\cos\left(\frac{\pi t}{12}\right),\ 0\le t\le 24,$ where $F(t)$ is measured in degrees Fahrenheit and t is measured in hours. (a) Sketch the graph of F on the grid below. (b) Find the average temperature, to the nearest degree Fahrenheit, between $t=6$ and $t=14$. (c) An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house? (d) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period? (a)  1: bell-shaped graph minimum 70 at $t=0$, $t=24$ only maximum 90 at $t=12$ only (b) Avg. $=\frac{1}{14-6}\int_6^{14}\left[80-10\cos\left(\frac{\pi t}{12}\right)\right]dt$ $=\frac{1}{8}(697.2957795)$ $=87.162$ or 87.161 $\approx 87°$ F 3 { 2: integral; 1: limits and $1/(14-6)$; 1: integrand; 1: answer; 0/1 if integral not of the form $\frac{1}{b-a}\int_a^b F(t)\,dt$ } (c) $\left[80-10\cos\left(\frac{\pi t}{12}\right)\right]-78\ge 0$ $2-10\cos\left(\frac{\pi t}{12}\right)\ge 0$ $\left.\begin{matrix}5.230\\ \text{or}\\ 5.231\end{matrix}\right\}\le t\le\left\{\begin{matrix}18.769\\ \text{or}\\ 18.770\end{matrix}\right.$ 2 { 1: inequality or equation; 1: solutions with interval }

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

(2.I) Determine and analyze the key features of exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.	<u>Concepts:</u>
	Parent Functions:
	• Exponential: $f(x) = a \bullet b^x$
	• Logarithmic: $f(x) = \log_b(x)$
	• Rational: $f(x) = \frac{1}{x}$
	• Polynomial: $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$
	• Power: $f(x) = a \bullet x^b$
	• Trigonometric: $f(x) = \sin(x)$ or $f(x) = \cos(x)$
	• Inverse Trigonometric $f(x) = \arcsin(x)$ or $f(x) = \arccos(x)$
	• Piecewise-defined functions
	• Key features
	  ○ Domain (interval notation*)
	  ○ Range (interval notation*)
	  ○ Symmetry (about x, y, origin)
	  ○ Maximum (local vs. absolute*)
	  ○ Minimum (local vs. absolute*)
	  ○ Extrema*
	  ○ Boundedness*
	  ○ Zeros
	  ○ Asymptotes
	  ○ Intervals of increasing (interval notation*)
	  ○ Intervals of decreasing (interval notation*)
	<u>Content:</u>
	Including, but not limited to:
	• Identify the parent function
	• Graph the function
	• Determine key features of the function
	• Analyze the function by the key features
		Below are the content and skill connections between the Algebra 2 TEKS and the AP standard.
		<u>Algebra 2 TEKS</u>
		(2A.2.A) graph the functions $f(x) = \sqrt{x}$, $f(x) = \frac{1}{x}$, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = \|x\|$, and $f(x) = \log_b x$ where b is 2, 10, and e, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum & minimum given an interval
		
		<u>AP Calculus CED Standards</u>
		• **FUN-3.E.1** The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists.
		• **FUN-3.E.2** The chain rule applied with the definition of an inverse function, or the formula for the derivative of an inverse function, can be used to find the derivative of inverse trigonometric functions.
		
		AP Calculus AB Released Multiple-Choice Problem (year unknown)
		What is the slope of the line tangent to the curve $y = \arctan(4x)$ at the point at which $x = \frac{1}{4}$?
		Ⓐ 2
		Ⓑ $\frac{1}{2}$
		Ⓒ 0
		Ⓓ $-\frac{1}{2}$
		Ⓔ -2

VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Geometry

- **G.6D** Verify theorems about the relationships in triangles, including proof of the Pythagorean Theorem, the sum of interior angles, base angles of isosceles triangles, mid-segments, and medians, and apply these relationships to solve problems.
- **G.8A** Prove theorems about similar triangles, including the Triangle Proportionality theorem, and apply these theorems to solve problems.
- **G.8B** Identify and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle, including the geometric mean, to solve problems.
- **G.9A** Determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems.
- **G.9B** Apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.
- **G.12B** Apply the proportional relationship between the measure of an arc length of a circle and the circumference of the circle to solve problems.
- **G.12C** Apply the proportional relationship between the measure of the area of a sector of a circle and the area of the circle to solve problems.
- **G.12D** Describe radian measure of an angle as the ratio of the length of an arc intercepted by a central angle and the radius of the circle.

Algebra 2

- **2A.2C** Describe and analyze the relationship between a function and its inverse (quadratic and square root, logarithmic and exponential), including the restriction(s) on domain, which will restrict its range.
- **2A.2D** Use the composition of two functions, including the necessary restrictions on the domain, to determine if the functions are inverses of each other.

Pre-Calculus

- **(2.G)** Graph functions, including ~~exponential, logarithmic~~, sine, cosine, ~~rational, polynomial, and power functions~~ and their transformations, including $af(x)$, $f(x)+d$, $f(x-c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems.
- **(2.H)** Graph $\arcsin x$ and $\arccos x$ and describe the limitations on the domain.
- **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational, polynomial, power~~, trigonometric, inverse trigonometric, ~~and piecewise defined~~ functions, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing.
- **(2.O)** Develop and use a sinusoidal function that models a situation in mathematical and real-world problems.
- **(2.P)** Determine the values of the trigonometric functions at the special angles and relate them in mathematical and real-world problems.
- **(4.A)** Determine the relationship between the unit circle and the definition of a periodic function to evaluate trigonometric functions in mathematical and real-world problems.
- **(4.B)** Describe the relationship between degree and radian measure on the unit circle.
- **(4.C)** Represent angles in radians or degrees based on the concept of rotation and find the measure of reference angles and angles in standard position.
- **(4.D)** Represent angles in radians or degrees based on the concept of rotation in mathematical and real-world problems, including linear and angular velocity.
- **(4.E)** Determine the value of trigonometric ratios of angles and solve problems involving trigonometric ratios in mathematical and real-world problems.
- **(4.F)** Use trigonometry in mathematical and real-world problems, including directional bearing.

AP Calculus AB/BC

- **LIM-1.E.2** The limit of a function may be found by using the squeezing theorem.
- **LIM-2.B.2** Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are continuous on all points in their domains.
- **FUN-3.A.3** The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions.
- **FUN-3.A.4** Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.
- **FUN-3.B.1** Derivatives of products of differentiable functions can be found using the product rule.
- **FUN-3.B.2** Derivatives of quotients of differentiable functions can be found using the quotient rule.
- **FUN-3.B.3** Rearranging tangent, cotangent, secant, and cosecant functions using identities allows differentiation using derivative rules.
- **FUN-3.C.1** The chain rule provides a way to differentiate composite functions.
- **FUN-3.E.1** The chain rule and definition of an inverse function can be used to find the derivative of an inverse function, provided the derivative exists.
- **FUN-3.E.2** The chain rule applied with the definition of an inverse function, or the formula for the derivative of an inverse function, can be used to find the derivative of inverse trigonometric functions.
- **FUN-4.A.1** The first derivative of a function can provide information about the function and its graph, including intervals where the function is increasing or decreasing.
- **FUN-4.A.2** The first derivative of a function can determine the location of relative (local) extrema of the function.
- **FUN-4.A.3** Absolute (global) extrema of a function on a closed interval can only occur at critical points or at endpoints.
- **FUN-4.A.4** The graph of a function is concave up (down) on an open interval if the function's derivative is increasing (decreasing) on that interval.
- **FUN-4.A.5** The second derivative of a function provides information about the function and its graph, including intervals of upward or downward concavity.
- **FUN-4.A.6** The second derivative of a function may be used to locate points of inflection for the graph of the original function.
- **FUN-4.A.7** The second derivative of a function may determine whether a critical point is the location of a relative (local) maximum or minimum.
- **CHA-3.D.1** The chain rule is the basis for differentiation variables in a related rates problem with respect to the same independent variable.
- **CHA-3.E.1** The derivative can be used to solve related rates problems; that is, finding a rate at which one quantity is changing by relating it to other quantities whose rate of change are known.

NOTE: These "essential knowledge" (EK) standards are from The College Board Concept Outline for AP Calculus AB/BC, not the TEKS.